Lec 19: 03/28/2012

Thin-Disk Spectrum:

In order to calculate the disk spectrum, we first need to know whether it is optically thin or thick. Since the optical depth is proportional to the column density, we can use the relation that was found in the previous lecture:

$$\nu \Sigma \sim \frac{\dot{M}}{3\pi}$$

Also:

$$\nu = \alpha C_s H = 2\alpha \left(\frac{R_g T}{\mu}\right)\left(\frac{R^3}{GM}\right)^{\frac{1}{2}}$$

This results in:

$$\Sigma \sim \alpha^{-1} \frac{\dot{M}}{6\pi}\left(\frac{\mu}{R_g T}\right)\left(\frac{GM}{R^3}\right)^{\frac{1}{2}} \sim 10^3 \left(\frac{\dot{M}}{10^{16}\,g\,s^{-1}}\right)\left(\frac{0.1}{\alpha}\right)\left(\frac{10^4 K}{T}\right)\left(\frac{M}{M_\odot}\right)^{\frac{1}{2}}\left(\frac{10^9 cm}{R}\right)^{\frac{3}{2}} g\,cm^{-2}$$

The optical depth in the perpendicular direction is:

$$\tau_\perp = \int_{-\infty}^{+\infty} n_{(z)}\, \sigma_T \, dz = \frac{\sigma_T}{\mu m_H}\int_{-\infty}^{+\infty} \rho_{(z)}\, dz = \frac{\sigma_T}{\mu m_H}\Sigma$$

Here m_H is the proton mass. Using the ~~values~~ of σ_T and

n_H, and the above expression for Σ, we find that $\tau_\perp \sim 10^3$. As a result, the thin disk is optically thick, which implies the emission must be blackbody.

We note that the temperature $T(R)$ is a function of R, and hence:

$D(R) = \sigma_B T_{eff}^4(R)$ (σ_B: Stefan-Boltzmann constant)

After using the expression derived for $D(R)$ (see the previous lecture), we have:

$$T_{eff}(R) = \left[\frac{3GM\dot{M}}{8\pi R^3 \sigma_B} \left[1 - \left(\frac{R_*}{R} \right)^{\frac{1}{2}} \right] \right]^{\frac{1}{4}}$$

Over most of the disk $R \gg R_*$, resulting in $T_{eff}(R) \propto R^{-3/4}$.

The intensity of the radiation at radius R is:

$$I_\nu(R) = \frac{\frac{2h\nu^3}{c^2}}{\exp\left[\frac{h\nu}{kT(R)}\right] - 1}$$

An observer at a distance D away, with a line of sight making an angle i relative to the symmetry axis of the disk, will measure a flux:

$$F_\nu = \int_{R_*}^{R_{out}} I_\nu \, d\Omega(R)$$

Where:

$$d\Omega(R) = \frac{2\pi R \, dR \cos i}{D^2}$$

Thus,

$$F_\nu = \frac{4\pi h\nu^3 \cos i}{c^2 D^2} \int_{R_*}^{R_{out}} \frac{R \, dR}{\exp\left[\frac{h\nu}{kT(R)}\right] - 1}$$

As usual, it is useful to find the asymptotic behavior of F_ν. In the Rayleigh-Jeans limit, $h\nu \ll kT(R_{out})$, which implies that:

$$F_\nu^{RJ} \propto \nu^2$$

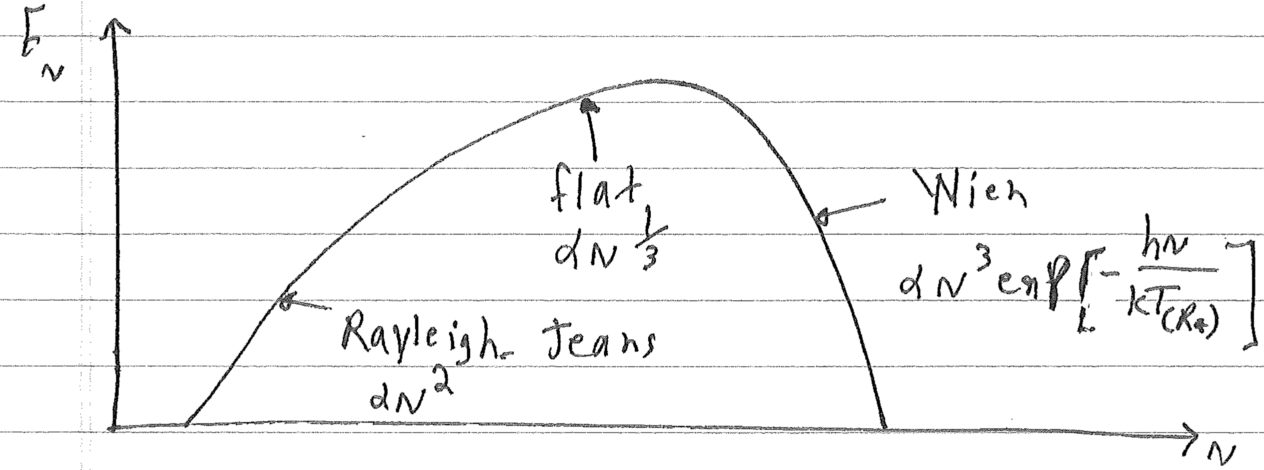
In the Wien limit, defined as $h\nu \gg kT(R_*)$, we have:

$$F_\nu^{W} \propto \nu^3 \exp\left[\frac{-h\nu}{kT(R_*)}\right]$$

In this limit the flux is dominated by the hottest portion of the disk near R_*. In between the two limits, we have:

$$F_\nu \propto \nu^{\frac{1}{3}} \int_0^\infty \frac{\eta^{\frac{5}{3}}}{e^\eta - 1} d\eta \propto \nu^{\frac{1}{3}} \quad , \quad \eta \equiv \frac{h\nu}{kT(R)} \simeq \frac{h\nu}{kT(R_*)} \left(\frac{R}{R_*}\right)^{-\frac{3}{4}}$$

In the Rayleigh-Jeans and Wien limits, the spectrum is that of a single blackbody, although at different temperatures. Between the two limits, the spectrum is a sum of blackbodies, which results in a $\propto \nu^{\frac{1}{3}}$ segment (so-called "flat segment"). The spectrum, shown below, can therefore be distinguished from those of other thermal sources by its stretched out appearance. It is usually not difficult to identify a spectrum belonging to a thin disk once it has been assembled from multifrequency observations. Note that the Wien region will give information about $T(R_*)$.

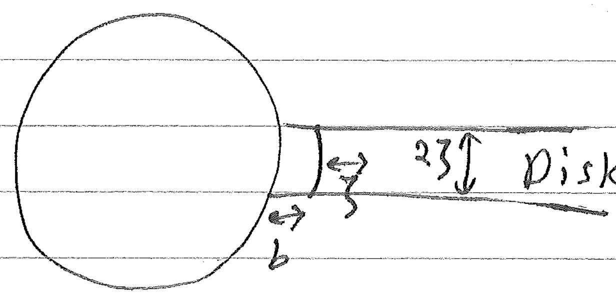


Boundary Layers:

Let us now study the role played in high energy astrophysics by the transition region between the inner edge of the disk and the surface of the compact object in more detail.

Since $v_R \ll v_\phi$, the angular velocity $\Omega(R)$ in the disk remains very close to its Keplerian value until the matter reaches just outside the surface of the star at $R = R_*$. Within a boundary layer of radial extent b, Ω must decrease from $\Omega_K(R_* + b)$ to $\Omega(R_*) < \Omega_K(R_*)$. For a very slow rotation (as compared with the Keplerian velocity) at R_*, the boundary layer must be in hydrostatic equilibrium in the radial direction;

$$\frac{1}{\rho}\frac{\partial P}{\partial \rho} \approx -\frac{GM}{R_*^2}$$



$P \sim c_s^2 \rho$

Thus;

$$\frac{c_s^2}{b} \sim \frac{GM}{R_*^2}$$

We also have hydrostatic equilibrium in the vertical direction;

$$\frac{1}{\rho}\frac{\partial P}{\partial z} \approx -\frac{GMz}{R_*^3} \Rightarrow \frac{P}{\rho \zeta} \sim \frac{GM\zeta}{R_*^3}$$

We therefore find;

$$\zeta \sim c_s R_* \left(\frac{R_*}{GM}\right)^{\frac{1}{2}} \Rightarrow b \sim \frac{\zeta^2}{R_*} = \left(\frac{\zeta}{R_*}\right)^2 R_* \ll R_*$$

The total luminosity produced by the disk is:

$$L_d = 2\int_{R_*}^{\infty} D(R)\, 2\pi R\, dR = \frac{3GM\dot{M}}{2}\int_{R_*}^{\infty}\left[1-\left(\frac{R_*}{R}\right)^{\frac{1}{2}}\right]\frac{dR}{R^2}$$

$$\Rightarrow L_d = \frac{GM\dot{M}}{2R_*}$$

On the other hand, the total gravitational energy released in the accretion is:

$$L_{acc} = \frac{GM\dot{M}}{R_*}$$

This implies that the power emitted from transition at the boundary layer is actually half of the total available power:

$$L_b = L_{acc} - L_d = \frac{GM\dot{M}}{2R_*}$$

The radiation emitted by the boundary layer is usually in the form of X-rays. It emerges through a region of radial extent $\sim \zeta$ on the two disk faces. This results in a blackbody radiation with temperature T_b, where:

$$4\pi R_* \zeta \sigma_B T_b^4 \approx \frac{GM\dot{M}}{2R_*}$$

This results in:

$$T_b \approx \left(\frac{GM\dot{M}}{8\pi R_*^2 \zeta \sigma_B}\right)^{\frac{1}{4}} \approx \left(\frac{R_*}{\zeta}\right)^{\frac{1}{4}} T_{(R_*)}$$

Using the following relation:

$$\zeta \simeq R_* \left[\frac{kT(R_*)}{\nu m_H}\right]^{\frac{1}{2}} \left(\frac{R_*}{GM}\right)^{\frac{1}{2}}$$

We find:

$$T_b \simeq \left[\frac{T_0}{T(R_*)}\right]^{\frac{1}{8}} T(R_*) \quad , \quad T_0 \equiv \frac{3}{8} \frac{GM\nu m_H}{kR_*}$$

In a neutron stare:

$$M \simeq M_\odot \quad , \quad R_* \simeq 10\,km \implies T_0 \simeq 3 \times 10^{11}\,K$$

Therefore:

$$T_b \simeq 3.6 \times T(R_*) \simeq 3 \times 10^{7}\,K$$

At this temperature, the boundary layer can produce a significant blackbody component above the regular disk spectrum. The combination of a stretched out thin disk spectrum plus a single (and hotter) blackbody component is indeed seen often in low mass X-ray binaries.